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# NOISE-INDUCED SNAP-THROUGH OF A BUCKLED COLUMN WITH CONTINUOUSLY DISTRIBUTED MASS: A CHAOTIC DYNAMICS APPROACH

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**Abstract**—For a spatially-extended dynamical system we illustrate the use of a chaotic dynamics approach to obtain criteria on the occurrence of noise-induced escapes from a preferred region of phase space. Our system is a buckled column with continuous mass, subjected to a transverse continuously distributed load that varies randomly with time. We obtain a stochastic counterpart of the Melnikov necessary condition for chaos—and snap-through—derived by Holmes and Mardsen for the harmonic loading case. Our approach yields a lower bound for the probability that snap-through cannot occur during a specified time interval. In particular, for excitations with finite-tailed marginal distribution, a simple criterion is obtained that guarantees the non-occurrence of snap-through. Copyright © 1996 Published by Elsevier Science Ltd.

## INTRODUCTION

The purpose of this paper is to illustrate the use of chaotic dynamics approach for obtaining criteria on the occurrence of noise-induced jumps in a spatially-extended dynamical system (i.e. a system governed by a partial differential equation with space and time coordinates). The system we choose for this illustration is a buckled column with continuous mass, subjected to a transverse continuously distributed force that varies randomly with time. The force may be due, for example, to seismic motion, pressures induced by air flow turbulence, or effects arising in hydrodynamical systems. Our approach is based on the use of the stochastic counterpart of the Melnikov function—the Melnikov process.

For a deterministic counterpart of our problem—a buckled column with uniform mechanical properties over its length, subjected to a transverse uniformly distributed load varying harmonically in time—a Melnikov-based necessary condition for the occurrence of snap-through was obtained by Holmes and Mardsen [1]. This condition is used in this paper as a building block for our extension of the Melnikov approach to the case of random transverse loading. The extension is effected in two steps. In the first step we explicitly or implicitly substitute for the random excitation process an approximation of that process by a sum of $N$ harmonic terms with random parameters, $N$ being a finite, albeit large number. The second step consists of using Wiggin's [2] extension of the Melnikov approach from the case of harmonic excitation to the case of excitation by a sum of $N$ harmonic terms.

In the following section we briefly review the Melnikov approach developed by Holmes and Marsden [1] for the continuous column with distributed harmonically fluctuating excitation. We then show that the Melnikov approach can be extended to the case of non-harmonic excitation, including random excitation, that is, the Melnikov necessary condition for chaos can be applied in this case. We apply our approach to a buckled continuous column excited by (a) broadband Gaussian noise and (b) dichotomous noise. For case (a) the probability that snap-through can occur during any specified time interval is always larger than zero, and if escapes may be viewed as rare events an upper bound for this probability is obtained in closed form. For case (b), a similar upper bound can be

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obtained numerically; in addition, the Melnikov necessary condition for chaos yields a simple criterion that guarantees the non-occurrence of snap-through. We present a numerical example whose results are in agreement with predictions based on that criterion. We then present our conclusions.

## EQUATION OF MOTION

Assume that (a) the mechanical properties of the column are uniform over its length, (b) the behaviour of the material is linearly elastic, (c) following the initial, static deformation of the column due to buckling the distance between the column supports is fixed, and (d) the column deformations are sufficiently small that, in the Taylor expansion of the projection of the elemental deformed column length on the line joining the column supports, terms of power higher than 2 can be neglected. The equation of the column is then [1, 3]

$$z_{tt} + z_{yyyy} + \left\{\Gamma - \xi \int_0^1 z_y^2(\zeta, t)\,\mathrm{d}\zeta\right\} z_{yy} = \varepsilon\{R(y, t) - \beta z_t\} \tag{1a}$$

$$R(y, t) = \gamma(y)\cos(\omega_0 t) + \rho(y) G(t) \tag{1b}$$

where the dimensionless deflection $z(y, t) = Z(Y, \tau)/\Delta$, $Z$ is the deflection at time $\tau$, $Y$ is the coordinate along the column length $\ell$, $y = Y/\ell$, $\Delta = Z_0(\ell/2)$ is the static deflection of the column $Z_0(Y)$ at coordinate $Y = \ell/2$, $t$ and $\tau$ are the dimensionless and dimensional time, respectively.

$$\Gamma = P_0\ell^2/EI, \tag{1c}$$

where $E$ is Young's modulus, $I$ is the moment of inertia of the column cross-section,

$$P_0 = P_{cr} + [EA/2\ell] \int_0^\ell (\mathrm{d}Z_0/\mathrm{d}Y)^2\,\mathrm{d}Y, \tag{1d}$$

$P_{cr} = k\pi^2 EI/\ell^2$ is Euler's critical buckling load, $k$ is a coefficient dependent upon the boundary conditions (for columns hinged at both ends $k = 1$), $A$ is the cross-sectional area, $\xi = \frac{1}{2}\Delta^2 A/I$, $\varepsilon\beta = c\ell^2/(mEI)^{1/2}$, $c$ is the viscous damping coefficient, $m$ is the column mass per unit length, $t = \omega_1\tau$ is the non-dimensional time, $\omega_1^2 = (EI/\ell^4 m)$, $\varepsilon\gamma(y) = f(Y)\ell^4 m/(EI\Delta)$, $f(Y)$ is the amplitude of the harmonic force per unit length, $G(t)$ is a non-dimensional non-periodic function, $\varepsilon\rho(y) = s(Y)\ell^4 m/(EI\Delta)$, $s(Y)$ is a measure of the non-periodic force per unit length. Both ends of the column are assumed to be hinged, that is, the boundary conditions are $z(0, t) = z(1, t) = z_{yy}(0, t) = z_{yy}(1, t) = 0$. The initial deflection $Z(Y, 0) = Z_0(Y)$. For our boundary conditions

$$Z_0(Y) = \Delta \sin(\pi Y/\ell). \tag{1e}$$

From equations (1c)–(1e) it follows that

$$\Gamma = \pi^2 + \pi^2\xi/2. \tag{1f}$$

The functions $\gamma(y)$ and $\rho(y)$ are expanded in the Fourier series

$$\gamma(y) = \gamma_0 + \sum_{n=1}^{\infty} \{\alpha_{\gamma on}\sin(n\pi y) + \beta_{\gamma on}\cos(n\pi y)\} \tag{2a}$$

$$\rho(y) = \rho_0 + \sum_{n=1}^{\infty} \{\alpha_{\rho n}\sin(n\pi y) + \beta_{\rho n}\cos(n\pi y)\}. \tag{2b}$$

## HARMONIC FORCING

The case $\rho(y) \equiv 0$ was studied by Holmes and Marsden [1]. We briefly summarize their results, to which we add expressions for the non-linear equations of motion obtained by using the Galerkin approach. The eigenvalues of the linearized, unforced equation are

$$\lambda_j = \pm\pi j(\Gamma - \pi^2 j^2)^{1/2}, \quad j = 1, 2, \ldots . \tag{3a}$$

From (1f) it follows that $\Gamma \geqslant \pi^2$. It was indicated earlier that we assume the deflections to be small. Therefore, from the expressions of $\Gamma$ and $\xi$ it follows that

$$\pi^2 < \Gamma < 4\pi^2. \tag{3b}$$

This means that the solution $z = 0$ has one positive and one negative eigenvalue and the system with $\varepsilon = 0$ and $\xi > 0$ has two non-trivial buckled equilibrium states. The system also has pure imaginary eigenvalues

$$\lambda_n = \pm \pi n(\Gamma - \pi^2 n^2)^{1/2}, \quad n = 2, 3, \ldots . \tag{3c}$$

The expansion of $z(y, t)$ in the eigenfunctions of the linearized problem

$$z(y, t) = \sum_{j=1}^{\infty} a_j(t) \sin(j\pi y), \tag{3d}$$

used with the Galerkin method, yields

$$\ddot{a}_j + \varepsilon\beta\dot{a}_j + (j\pi)^2 \left\{ (j\pi)^2 - \left[ \Gamma - (\xi\pi^2/2) \sum_{k=1,2,\ldots} k^2 a_k^2 \right] \right\} a_j = 2\varepsilon\phi_j \cos(\omega_0 t), \tag{4}$$

where $\phi_j = \int_0^1 \gamma(y) \sin(j\pi y)\, \mathrm{d}y$.

The Melnikov function for the harmonically excited system can be written as

$$M(t) = \int_{-\infty}^{\infty} \int_0^1 [R(y, \theta)\dot{z}_0(y, \theta - t) - \beta \dot{z}_0^2(y, \theta - t)]\, \mathrm{d}y\, \mathrm{d}\theta \tag{5}$$

where $R(y, t)$ is given by (1b) and $\rho(y) \equiv 0$, and the homoclinic orbit of the unperturbed system has coordinates $(z_0, \dot{z}_0)$. Using symplectic forms, Holmes and Marsden [1] derived the result

$$z_0(y, t) = (2)^{1/2} \sin(\pi y) \mathrm{sech}[t\pi(\Gamma - \pi^2)^{1/2}] \tag{6a}$$

$$\dot{z}_0(y, t) = -(2)^{1/2}\pi(\Gamma - \pi^2)^{1/2} \sin(\pi y) \mathrm{sech}[t\pi(\Gamma - \pi^2)^{1/2}] \tanh[t\pi(\Gamma - \pi^2)^{1/2}]. \tag{6b}$$

An alternative way to obtain Eq. (6) is to note that the unperturbed counterpart of system (4) has a fixed point at $(a_1, a_1, a_2, a_2, \ldots) = (0, 0, 0, 0, \ldots)$. Assume $a_j(t) = a_j(t) \equiv 0$ $(j = 2, 3, \ldots)$. The unperturbed counterpart of the system (4) then reduces to the unperturbed Duffing–Holmes equation

$$\ddot{a}_1 + (\pi)^2\{(\pi)^2 - [\Gamma - (\xi\pi^2/2)a_1^2]\} a_1 = 0. \tag{7a}$$

The solution of the unperturbed system with initial conditions at the fixed point is therefore

$$a_1(t) = (2)^{1/2} \mathrm{sech}[t\pi(\Gamma - \pi^2)^{1/2}] \tag{7b}$$

$$a_j(t) = \dot{a}_j(t) \equiv 0 \quad (j = 2, 3, \ldots), \tag{7c}$$

the latter equations being consistent with the fact that, by virtue of (3b) and (3c), the system's eigenvectors are contained in the plane $a_1, \dot{a}_1$. Equation (6a) then follows from (3d).

We now consider (4) for the particular case $\gamma(y) = \gamma_c$, $\phi_j = 2\gamma_0/(\pi j)$, $j = 1, 3, \ldots$, and $\phi_j = 0$, $j = 2, 4, \ldots$ . Provided that the non-resonance condition $\omega_0^2 \neq -\lambda_j^2$ holds, (4) has unique solutions of $O(\varepsilon)$. If the non-resonance condition were violated, the linearized counterparts of (4) would have solutions of $O(1)$. This would violate a basic assumption of Melnikov theory [1].

After inserting (6b) into (5),

$$M(t) = k_1\beta + [\alpha_{\gamma_0 1}/2 + 2\gamma_0/\pi]\, k_2(\omega_0) \sin(\omega_0 t) \tag{8a}$$

$$k_1 = -(2/3)\pi(\Gamma - \pi^2)^{1/2} \tag{8b}$$

$$k_2(\omega_0) = -(2\omega_0/\pi)/(\xi^{1/2})\, \mathrm{sech}\{\omega_0/[2(\Gamma - \pi^2)^{1/2}]\}. \tag{8c}$$

For sufficiently small $\varepsilon$, the stable and unstable manifolds of the perturbed system—which emanate in forward and reverse time, respectively, from the one-dimensional torus associated with the saddle point of the unperturbed system—intersect transversely if $M(t)$ has simple zeros. The dynamics of the system then contains a horseshoe, which is associated with the possible existence of a strange attractor.

## NON-HARMONIC AND RANDOM FORCING

In this section we summarize and extend to our continuous system the approach used in Frey and Simiu [4], Simiu and Hagwood [5] and Sivathanu *et al.* [6] to apply the Melnikov approach to dynamical systems with non-harmonic or random excitation.

### *Quasiperiodic excitation*

Assume that $\rho(y) \neq 0$ and the function $G(t)$ consists of a sum of harmonics, that is,

$$G(t) \equiv G_{\mathrm{qp}}(t) = (1/d) \sum_{i=1}^{N} d_i \cos(\omega_i t - \psi_i), \quad d \neq 0. \tag{9}$$

The Galerkin method yields equations with the same left-hand side as (4), and right-hand side consisting of a sum of $N$ terms, each of which is similar to the right-hand side of (4). The Melnikov function is also given by (5), in which $R(y, t)$ is given by (1b) and $G(t)$ by (9) [2]. Provided that the non-resonance condition is satisfied for each of the frequencies $\omega_i$ ($i = 0, 1, 2, \ldots, N$),

$$M(t) = k_1 \beta + [\alpha_{\gamma_0 1}/2 + 2\gamma_0/\pi] k_2(\omega_0) \sin(\omega_0 t) + [\alpha_{\rho 1}/2 + 2\rho_0/\pi] \sum_{i=1}^{N} (d_i/d) k_2(\omega_i) \sin(\omega_i t - \Psi_i). \tag{10}$$

Following Wiggins [2], we refer to the Melnikov function for the quasiperiodic excitation cases as the generalized Melnikov function. For a one-degree-of-freedom quasiperiodically-excited system it has been proved that, for chaotic behaviour to be possible, the generalized Melnikov function must have simple zeros [2]. The proof applies with no modification to our case, yielding the result that for our quasiperiodically-excited system to behave chaotically, its generalized Melnikov function must have simple zeros.

### *Function* G(t) *approximately expressible as a quasiperiodic function*

A similar Melnikov necessary condition for chaos holds if $G(t)$ can be approximated as closely as desired by a sum of $N$ harmonics with amplitudes of order $(\Delta\omega)^{1/2}$, where $\Delta\omega$ is a small frequency interval. For example, consider a function $G(t) \equiv G_{\mathrm{F}}(t)$ that has a Fourier transform. Then $G(t)$ can be approximated sufficiently closely by a sum of $N$ harmonics with amplitudes of order $(\Delta\omega)^{1/2}$, where $N$ is sufficiently large ($\Delta\omega$ is sufficiently small). The generalized Melnikov function corresponding to this quasiperiodic sum is approximated sufficiently closely by (5), where $R(y, t)$ has the expression of (1b) with $G(t) \equiv G_{\mathrm{F}}(t)$. It follows from the extension of Melnikov theory to quasiperiodically-excited systems that the behaviour of the system can be chaotic if the generalized Melnikov function so obtained has simple zeros.

We now consider a function $G(t) \equiv G_{\mathrm{nF}}(t)$ for which no Fourier transform can be defined. Assume, however, that a function $G_1(t)$ exists that has a Fourier transform, and that

$$\int_{-\infty}^{\infty} \int_0^1 [G_{\mathrm{nF}}(t) - G_1(t)]\, \dot{z}_0(y, \theta - t)\, \mathrm{d}\theta\, \mathrm{d}y < \delta \tag{11}$$

where $\delta$ is as small as desired. The error in the calculation of $M(t)$ due to the substitution for $G_{\mathrm{nF}}(t)$ of the function $G_1(t)$—or of a sum of harmonics approximating $G_1(t)$ sufficiently closely—is then as small as desired. Therefore, in this case the generalized Melnikov function can be calculated by using (5), where $R(y, t)$ is given by (1b) and $G(t) \equiv G_{\mathrm{nF}}(t)$. Again, the system behaviour can be chaotic if the generalized Melnikov function so obtained has simple zeros. An example is given in a subsequent section.

Recall that, for harmonic and quasiperiodic excitation, the non-resonance condition $\omega^2 \neq -\lambda_j^2$ has to be satisfied so that the solutions $a_{Lj}$ ($j = 1, 2, \dots$) of the linearized counterparts of the Galerkin equations be of order $O(\varepsilon)$, rather than of $O(1)$. However, if $G(t)$ has a time-domain Fourier transform [or a function $G_1(t)$ exists such that the quantity given by (11) is as small as desired], then $G(t)$ [or $G_1(t)$] has components over a continuous range of frequencies, including elemental components with frequencies equal to the natural frequencies of the system. In this case it can be shown that the solutions $a_{Lj}$ are of $O(\varepsilon^{1/2})$ (see, e.g. Meirovich [7] or Simiu and Scanlan [8], pp. 546 and 197). For sufficiently small $\varepsilon$ the solutions $a_{Lj}$ will therefore be as small as desired, and non-resonance conditions are not required for the assumptions of Melnikov theory to be satisfied. This remains true if the function $G(t)$ is approximated by a sum of harmonic terms with amplitudes proportional to $(\Delta\omega)^{1/2}$, provided that the interval $\Delta\omega$ is sufficiently small (or, equivalently, that $N$ is sufficiently large). However, if $\gamma(y) \neq 0$, the non-resonance condition must be satisfied for the harmonic excitation with frequency $\omega_0$.

*Random forcing and Melnikov processes*

A vast class of stochastic processes can be closely approximated by sums of large numbers of periodic terms with random parameters. Conditional upon having occurred, each realization of a stochastic process may be viewed as a sum of periodic terms with fixed parameters. The results discussed earlier for systems with quasiperiodic excitation are therefore applicable to each of the realizations of the stochastically excited system. Each realization of the excitation process induces a generalized Melnikov function. Solutions of the system excited by that realization can be chaotic if the corresponding generalized Melnikov function has simple zeros. The ensemble of generalized Melnikov functions induced by the excitation process is referred to as the system's Melnikov process. Just as statements on the behaviour of deterministic systems can be made by considering the behaviour of their Melnikov functions (or generalized functions), so too probabilistic statements on the behaviour of stochastic dynamical systems can be made by considering the probabilistic behaviour of their Melnikov processes. The Melnikov process can be obtained by using (5), where $R(y, t)$ is given by (1b) and $G(t)$ is a random excitation process. For details on this Melnikov-based approach to the study of stochastic differential equations with Gaussian noise, shot noise, dichotomous noise, or other types of non-Gaussian noise, see [4–6, 9, 10].

## GAUSSIAN EXCITATION

We now assume that $\gamma(y) \equiv 0$ and the excitation $G(t)$ is a Gaussian process with spectral density $\Psi(\omega)$. Owing to the linearity of the expression for the Melnikov process, the latter is Gaussian with spectral density, mean and variance [4]

$$\Psi_M(\omega) = \Psi(\omega) k_2^2(\omega), \tag{12a}$$

$$E(M) = k_1 \beta, \tag{12b}$$

$$Var(M) = [\alpha_{\rho 1}/2 + 2\rho/\pi]^2 \int_0^\infty \Psi(\omega) k_2^2(\omega)\, d\omega. \tag{12c}$$

The fact that the Melnikov process is normally distributed means that, over any finite time interval $T$, there is a finite probability that the process will have simple zeros no matter how small the noise. On the other hand, if from time $t = 0$ to $t = T$, the Melnikov process being considered has no zero upcrossings, then no transitions from motions on one side of the undeformed shape of the column to motions on the other side (i.e. no snap-through) may be expected to occur during the time interval $T$.

For sufficiently large ratio $k = E(M)/[Var(M)]^{1/2}$, it can be shown that the probability that the Melnikov process has simple zeros during time interval $T$ is [11]

$$P_T = 1 - \exp[-E(k)T], \quad E(k) = \nu \exp(-k^2/2), \tag{13a, b}$$

$$\nu = (1/2\pi)\left\{\left[\int_0^\infty \omega^2 \Psi_M(\omega)\,d\omega\right]\Big/\left[\int_0^\infty \Psi_M(\omega)\,d\omega\right]\right\}^{1/2}, \tag{14}$$

where $\nu$, the mean zero upcrossing time of the Melnikov process, is a lower bound for the mean time between transitions, and $p_T$ is a lower bound for the probability that no transition occurs during time $T$; for details on a similar application, see [12].

## DICHOTOMOUS NOISE

We assume that $\gamma(y) \equiv 0$ and the excitation consist of dichotomous coin-toss square-wave noise with ordinate $\rho_0$, uniformly distributed over the length of the column, that is

$$G(t) = c_n \quad [\alpha + (n-1)]t_0 < t \leqslant (\alpha + n)t_0, \tag{15}$$

where $n = \ldots, -2, -1, 0, 1, 2, \ldots$ . is the set of integers, $\alpha$ is a random variable uniformly distributed between 0 and 1, $c_n$ are independent random variables that take on the values $-1$ and 1 with probabilities 1/2 and 1/2, respectively, and $t_0$ is a parameter of the process $G(t)$ [5, 6, 13].

A rectangular pulse wave of amplitude $c_n$ and length $t_0$ centred at coordinate $t_n = (\alpha + n - 1/2)t_0$ had Fourier transform [14]

$$F_n(\omega) = c_n |(2/\omega)\sin(\omega t_0/2)\exp(-j\omega t_n)|.$$

The pulse itself can therefore be expressed as a sum of harmonic terms approximating as closely as desired the inverse Fourier transform of $F_n(\omega)$. Each realization of the coin-toss dichotomous square-wave can be approximated arbitrarily closely by a finite superposition of such sums. The Melnikov approach can then be applied to the system excited by the approximating noise process. The expression for the Melnikov function follows from (5) (see end of section *Random forcing and Melnikov processes*) and (6b)

$$M(t) = k_1\beta + [2\rho_0/\pi](2)^{1/2}\int_{-\infty}^{\infty} G(\theta)\,\{\pi(\Gamma - \pi^2)^{1/2}\operatorname{sech}[\pi(\Gamma - \pi^2)^{1/2}(\theta - t)]$$
$$\times \tanh[\pi(\Gamma - \pi^2)^{1/2}(\theta - t)]\}\,d\theta \tag{16}$$

or

$$M(t) = -(2/3)(\pi\xi^{1/2}/2^{1/2})\beta + [2(2)^{1/2}/\pi]\rho_0 F(t) \tag{17}$$

where $\rho_0$ is the ordinate of the dichotomous noise, and $F(t)$ is the integral in (17). The Melnikov necessary condition for the occurrence of escapes is

$$\rho_0 > \{\pi^3\xi/6\max[F(t)]\}\beta. \tag{18}$$

The maximum possible absolute value of $F(t)$, attained when all $c_n$ are positive (negative) for $\theta > 0$ and negative (positive) for $\theta < 0$, is immediately found to be 2. Therefore, from (18), snap-through cannot occur if

$$\rho_0 \leqslant \rho_{0M} = 2.584\xi^{1/2}\beta. \tag{19}$$

This condition, guaranteeing the non-occurrence of snap-through, is weak because (a) it is based on the Melnikov condition for chaos, which is necessary, but not sufficient, and (b) it is based on the upper bound for $|F(t)|$.

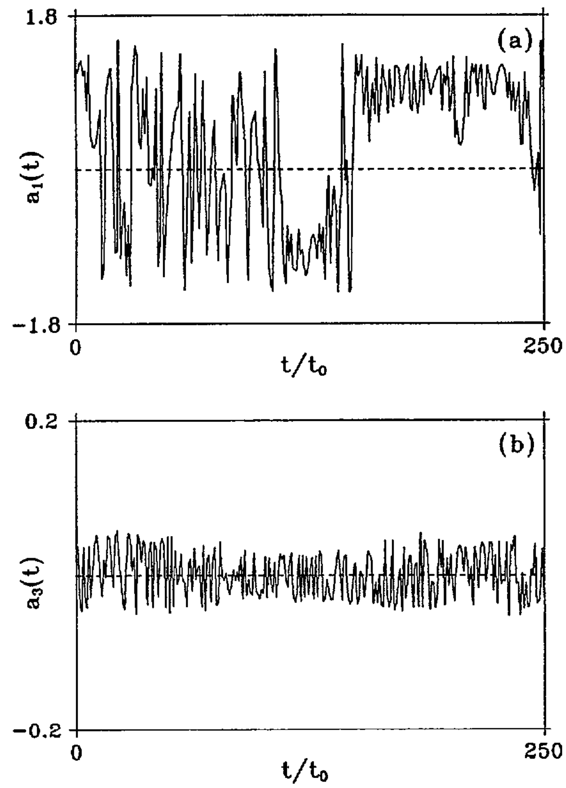


Fig. 1. Examples of steady-state time histories of amplitudes $a_1(t)$ and $a_3(t)$, dichotomous excitation.

## NUMERICAL EXAMPLES

We consider Equation (1), $\gamma(y) \equiv 0$, $\rho(y) = \rho_0$, $\ell = 0.45$ m, and assume the column cross-section is rectangular with dimensions $h = 0.0005$ m, $b = 0.0125$ m, $E = 200{,}000$ MPa, $\Delta = 0.0005$ m, $\varepsilon = 0.1$, $c = 1.365 \times 10^{-3}$ kg/m/s, and $t_0 = 0.2$. We have $A = 6.25 \times 10^{-6}$ m$^2$, $I = 1.30208 \times 10^{-13}$ m$^4$, $m = 0.04875$ kg/m, $\beta = 7.76 \times 10^{-2}$, $\xi = 6.0$. From (19), $\rho_{0M} = 048$.

The equations of motion were solved numerically for given realizations of (15) and various values of the excitation amplitude $\rho_0$. For the parameters just listed the smallest excitation for which snap-through was observed was $\rho_{0,\min} \approx 1.4 > 0.48$. Note that $\rho_{0,\min}$ depends on $t_0$, which is a measure of the average periodicity of the random excitation. For example, for $t_0 = 1.0$, all other parameters being unchanged, $\rho_{0,\min} \approx 14$, that is, the dichotomous noise is less effective in inducing snap-through if $t_0 = 1$ than if $t_0 = 0.2$.

For the parameters listed earlier, except for $t_0 = 1.0$, a steady-state time history of the amplitudes $a_1(t)$ and $a_3(t)$ for the first and third Galerkin modes is shown in Fig. 1; Fig. 2 shows the evolution in time of the column shape at snap-through $[z(y, t_n) = z_1(y, t_n) + z_3(y, t_n); \quad z_j(y, t_n) = a_j(t_n)\sin(\pi j y)]$.

## CONCLUSIONS

We examined a spatially-extended system consisting of a column with continuous mass distribution excited by a uniformly distributed load varying randomly in time. Following a discussion of the non-resonance condition for stochastic excitation, we extended Holmes and Marsden's [1] expression for the Melnikov function induced by harmonic loading to the case of stochastic loading. By solving numerically the Galerkin equations for the columns subjected to a uniformly distributed dichotomous load process, we verified for this case the validity of the stochastic Melnikov condition for non-occurrence of snap-through. We also verified that snap-through does occur for excitations sufficiently larger than the excitation yielded by the Melnikov necessary condition for chaos.

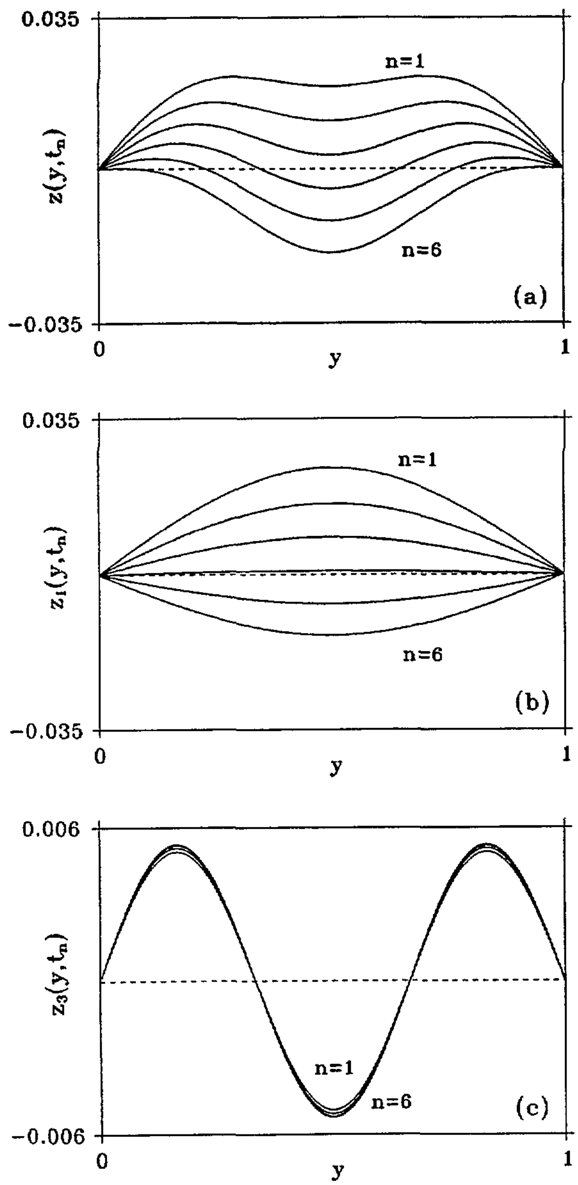


Fig. 2. Evolution in time of column shapes $z(y, t_n)$ and $z_j(t)$ ($j = 1, 3$) at snap-through ($t_n = nt_0/300$).

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